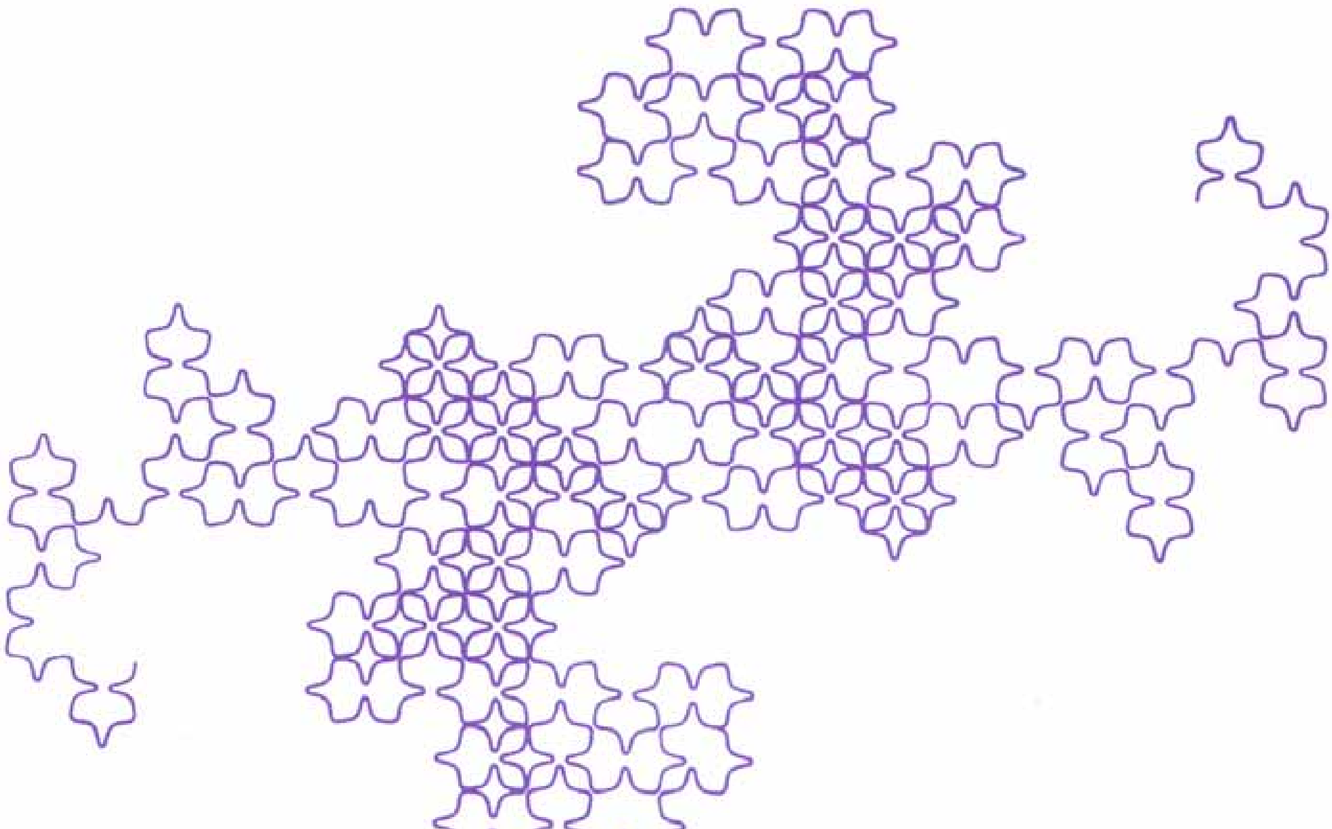
MT 128



MATHEMATICS TEACHING · SEPTEMBER 1989

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MT 128

The ‘ones‘ on p‘7l ofMTl27 and on p2 of this MT were designed by
Patrick Keane
Artwork *Zena Ginifer*

MATHEMATICS AND MODERN ART: COMPUTER GRAPHICS

Ulrich Grevsmühl

First initiatives on the way to computer art were made by scientists and mathematicians who had the knowledge and the necessary apparatus available for producing aesthetic programmes, in North America and West Germany more than thirty years ago. Fascinated by the appeal of Lissajous‘ figures many experiments with mechanical and electrical oscillating systems were carried out, at first by employing mechanical plotters, later by the use of analog computers in the form of oscilloscopes. The first large exhibition of electronically generated graphics was shown in the USA in 1953 where photographs taken by Ben F. Laposky of oscillographs were presented.

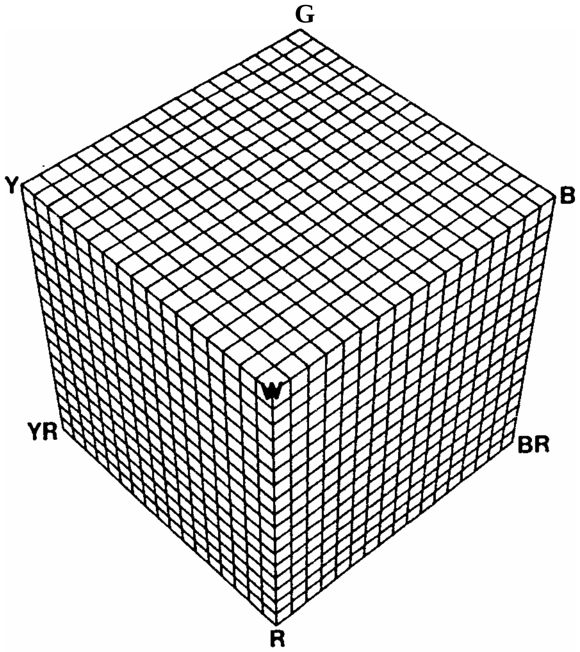
In Europe an exhibition with a similar theme showing works of Herbert W. Franke was organized in 1959. In the sixties advances in computer art were made especially by a group under Max Bense in Stuttgart where the works of pioneers like Frieder Nake and Georg Ness reached international recognition. In the rapidly developing field computer artists at that time began to make use of digital computers, display monitors, fast printers, photocopies and computer controlled photographs, slides and films. New techniques were developed and perfected over the years. Today we find computer generated graphics, methods of picture processing, computer automated design, computer simulation and animation which also take in many fields of traditional art like sculptures, films and videos, choreography, poems and music [1]. One of the recent achievements, as demonstrated in the images of fractal geometry, has been that it was not only possible to create 3-D pictures of imaginary landscapes and other scenes in perspective but also to include the reality of time change and movement [2]. There have been, and there still are, many discussions about whether or not it is at all possible for art to be created with the help of a machine. Is computer art really art? When one considers the various points of view, it has to be borne in mmd that computer generated graphics, for instance, which is dealt with in the following examples, is not a style of art but rather a method of employing certain instruments. It is not relevant what the artist can do with the computer but rather what messages, ideas and feelings can be transported with this medium. New technologies mean both new possibilities for creating works of art and also new messages; both a new syntax and a new semantics. For this reason the history of computer art is closely related to the development of the technology and one can be curious about what new media of communication will be invented in the future.

BECK & JUNG

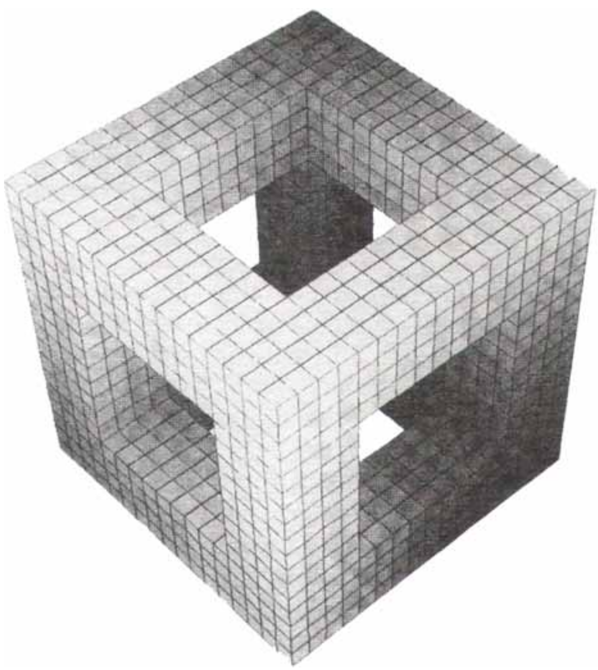
Both born in Lund, Sweden, 1939. Live in Lund.

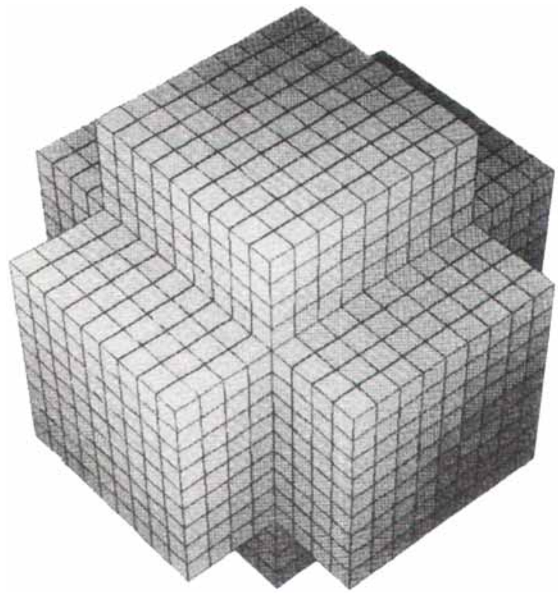
Holger Bäckström and Bo Ljungberg have been working together since the mid-sixties under the name Beck & Jung and belonged to the first group of artists who turned to computer technology for new inspirations. They started to make computer pictures in colour in 1972 with the help of the *Colour Ink Jet Plotter* which had been developed at the University of Lund. This plotter can plot points, lines and areas. A plot-head containing a jet nozzle for each of the three primary colours yellow, red and blue is used to produce 17,576 different colour shades in a special raster system. lt can colour more than 100,000 separate points per second, 25 points per square millimeter, each point having a diameter of 0.2 millimeter.

Since 1980 Beck & Jung have investigted the world of colours by applying the Colour Ink Jet Plotter to their *Chrorno Gube.* This cube is a model for the systematization of colours. The colours black (BL) and white (W) are placed in the opposite vertices of a cube. Then the primary colours red. (R), yellow (Y) and blue (B) are arranged in the adjacent corners around the white pole (W), and correspondingly green (G), orange (YR) and violet (BR) around the black pole. By this all complementary colours, ie red and green, are situated in opposite vertices of the cube. The edges of the cube are divided into 17 parts each. The total cube is therefore built up from 4913 (= 17^3) small cubes which contain all conceivable colour shades between the various colour poles. Thus the total cube contains our whole world of colours [3].



Activity 1: Permutational combinations
Beck & Jung have created many versions from the Chromo Gube by removing or moving some of the 4913 small cubes. Two of these, *Hole in One (1980)* and *Puno (1980)*, are reproduced here, unfortunately in black and white only. Show that the total number of versions is 24913.



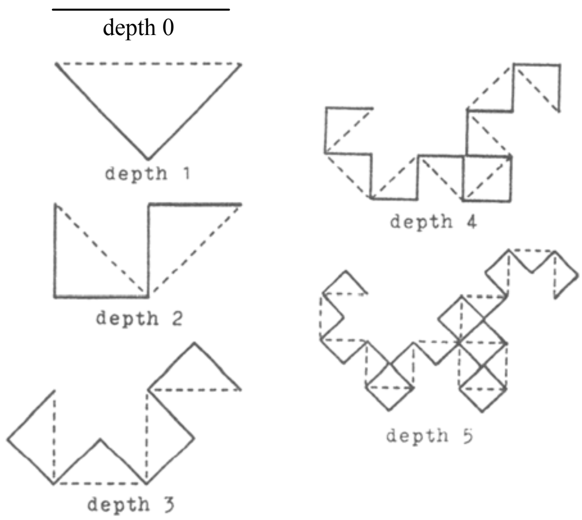


Born in Vienna, 1927. Lives in Puppling near Munich

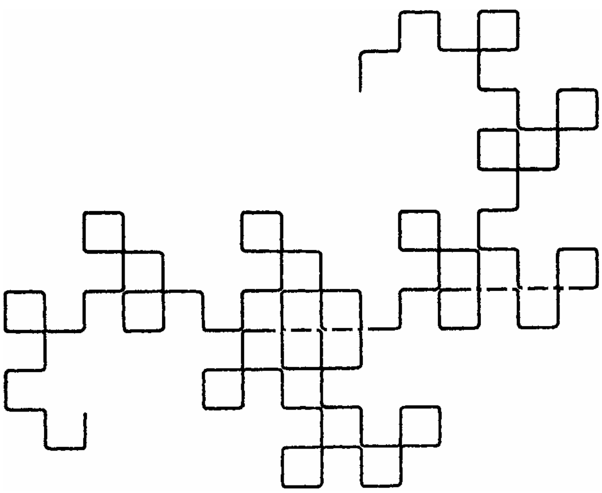
Working as a free-lance journalist, Franke established himself from 1961 as Germany‘s best known science fiction author. Since 1970 he has been producing computer graphics on digital systems and is today one of the foremost advocates of technically assisted art. One of his research interests is concerned with rational aesthetics and the kybernetic theory of art. In this theory art is treated as an information process: a work of art may be considered as a form of communication with a certain amount of information and redundance through which emotions are created.
Models of kybernetic aesthetics try to explain the beauty of a work by taking into account both the amount of available information and the mechanisms of perception. lt appears that an information flow of up to 16 bits per second may still be registered by our consciousness and is linked to one‘s interest whereas a larger flow of information results in confusion; a smaller one in boredom. Computer generated graphics, and in particular dragon functions, are suitable to test models of kybernetic aesthetics since the complexity of the structure may successively be increased when building up the graphics. They include at the same time the fundamental principles of aesthetics and art. In ***Drakula (1971)*** Franke has used the dragon function as the underlying structure for generating computer graphics *[4,5]*.

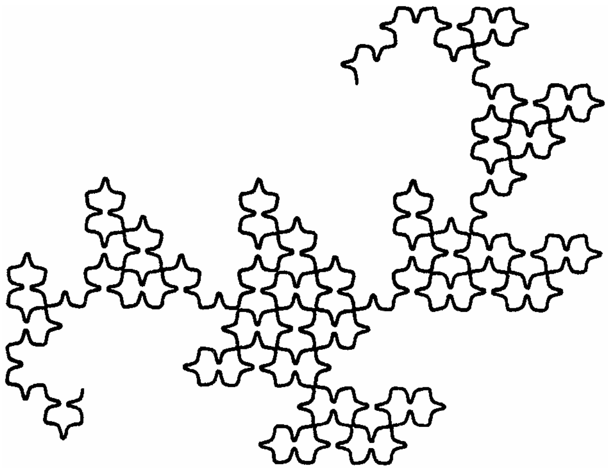
Activity 2: Dragon graphs
The dragon function received its name from the shape of its graphs which look like stylised dragons. Bach graph is generated by a recursive process and arises from its predecessor by replacing the sides by

right-angled ,V‘-shapes in alternating orientation. Consequently, the sides are shortened by a factor of 1/√2. As each graph consists of two congruent parts which can be mapped onto one another by an anticlockwise rotation of 90° around the centre of the graph, a paper folding method suggested by the physicist J E Heigway [6] may be applied to produce the shapes of the first few dragon graphs but not the actual length of the sides. In this method a strip of paper is repeatedly halved by folding in the same direction. If after n folds the paper strip is unfolded and angles of 90° are established in each of the corners, a dragon graph of depth n will be formed. Write a recursive computer program which generates dragon graphs of given depths.



These three graphics by H W Franke are all based on a mirrored version of the dragon graph of depth 7. Whereas the first one shows the graph itself but with the last side of the last ,V‘-shape missing, in the second one the straight lines are replaced by curved elements. In the third graphic ***Drakula 12A/1971*** (see front cover of this ***MT),*** so-called super-signs arc created by superimposing on the original second graph another one which has been rotated by 180°.





MANFRED MOHR

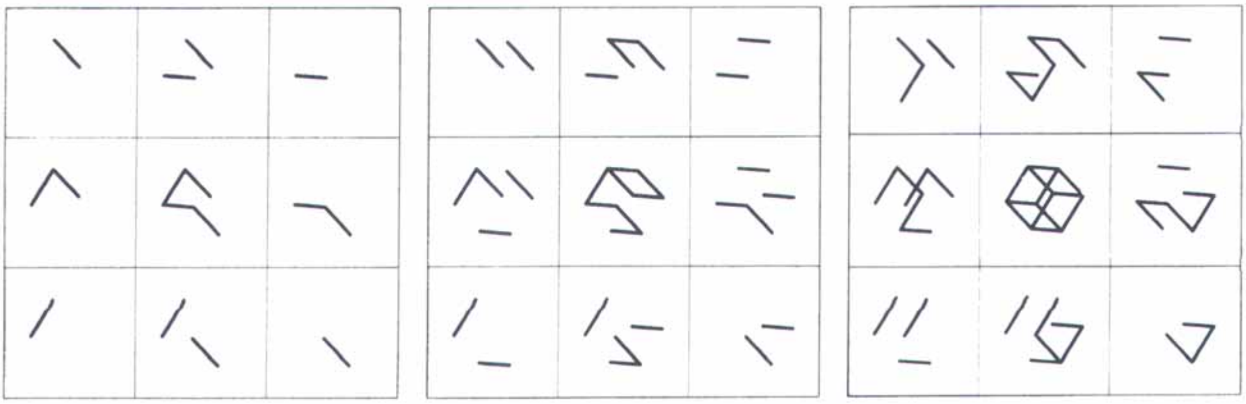
Born in Pforzheim, West Germany, 1938. Lives in New York.

Manfred Mohr started to employ the computer and the plotter as tools for carrying out his works in 1969 when he gained access to the computer centre of the Meteorological Institut in Paris during his years as an arts student. Since then he has developed various algorithms for generating two-dimensional iconic signs [7].

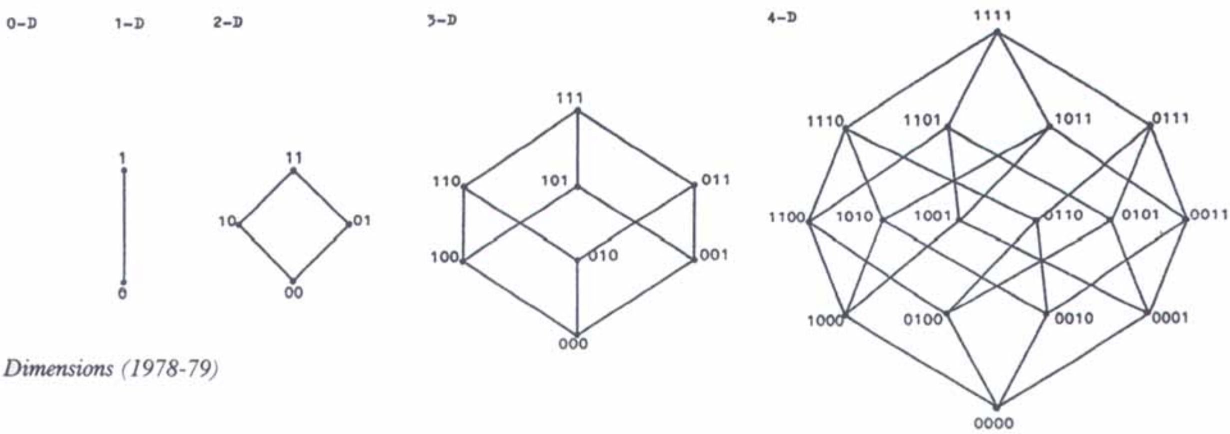
Activity 3: Generation of être-graphiques
Consider an affine projection of a skeletal cube. If the 12 edges of the cube arc taken away one by one, the 3-D illusion will gradually dissolve and 2-D signs, so-called *être-graphiques,* will be generated. Find the number of combinations for the cases that 0,1,2,... or all 12 edges arc missing and show that there are 4096 possible être-graphiques which can be obtained from a cube this way. Write a computer program to draw an affine projection of a cube which systematically generates all 4096 être-graphiques A few of these are depicted in Mohrs‘ work *P-185-D (1976, ink on paper, 3 parts, 40 x 40 cm each)* [8].

In his work phases *Cubic limit* and *Divisibility* Mohr has extensively investigated the skeletal strueture of the cube as a generator of 2-D signs by applying various methods and procedures. This work was extended in *Dimensions (1978-79, 1 987-88)* to the 4-D hyper-cube where at first the graph, later 4-D rotations were used to generate signs and shapes
[7,9].

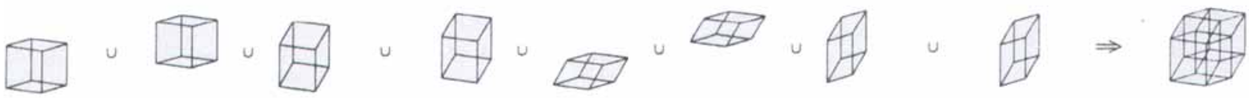
Cartesian objects of various dimensions like the point (0-D), the straight line (1-D), the square (2-D), the cube (3-D) and the 4-D hyper-cube can be represented by two-dimensional graphs where the binary numbers of the vertices account for the spatial relationships: a change in digit indicates a change in dimension



P-185-D (1976)



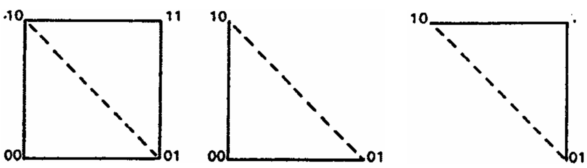
Dimensions (1978-79)

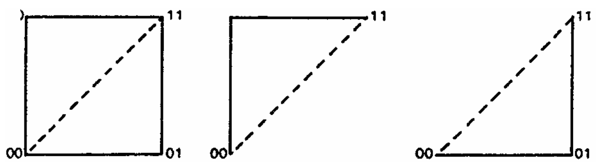


Dimensions (1987-88)

The construction of a 4-D hyper-cube is based on a set of eight cubes put together in such a way that each edge of a cube simultaneously belongs to three different but adjacent cubes. Note that the spatial representation of the 4-D hyper-cube is topologically equivalent 10 its graph.

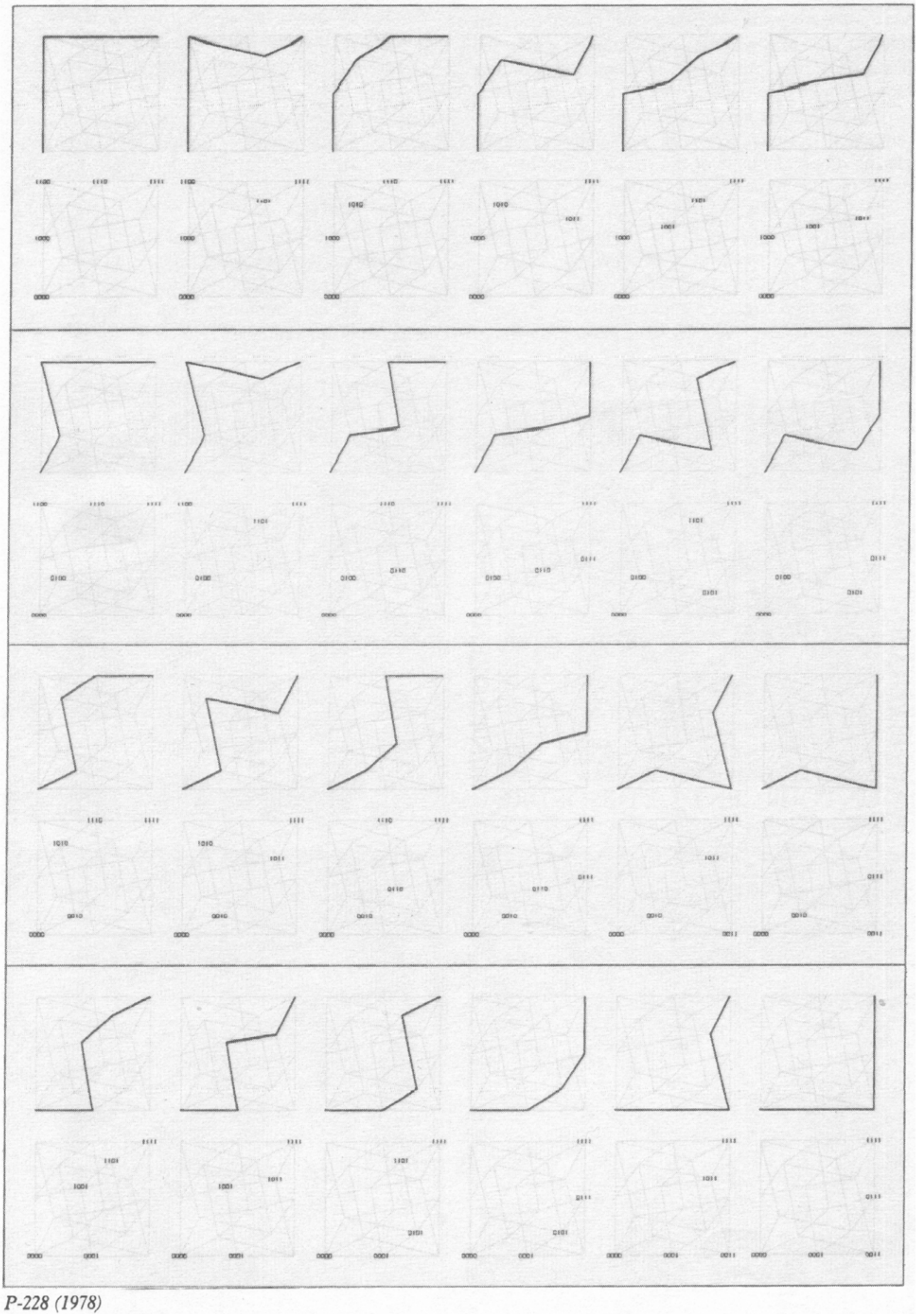
Activity 4: Diagonal paths In the graph of a cartesian object a diagonal path is by definition a path which leads from one point (vertex) to the diagonally opposite point passing through each of the given dimensions just once. In two dimensions (a square) there are two diagonals, each having two diagonal paths.





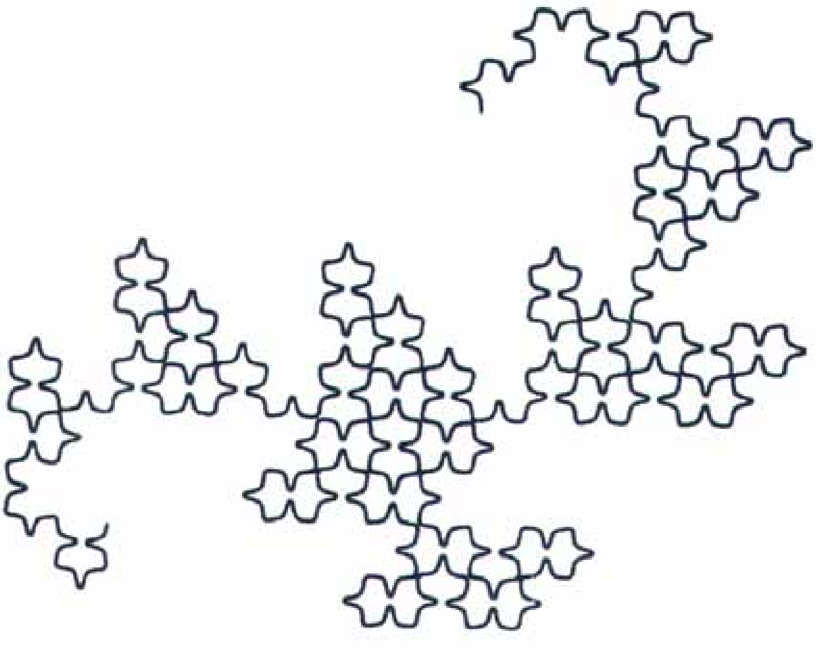
In three dimensions (a cube) there are 4(= 22) diagonals, each having 6 (= 3 x 2) diagonal paths. In four dimensions (a hypercube) there are 8 $(=2^2)$ diagonals, each having 24 (= 4 x 3 x 2) diagonal paths. These 192 paths have been drawn in Mohr's work P228 (1978, ink on paper, 8 parts, 55 x 4l cm each. Note that for this purpose the hexagonal shape of the graph of the 4-D hyper-cube has been transformed into square format whilst maintaining the inherent properties of the structure. Here only the 24 paths belonging to the diagonal 0000-1111 are depicted where the binary numbers indicate the passage through the four dimensions [9].

Consider the graphs of a square, a cube, and a 4-D hyper-cube. With the help of a computer program find for each of the diagonals all the possible diagonal paths by calculating the changes in binary digits and drawing the corresponding paths. For a 4-D hyper-cube, for instance, the diagonals are given by 0000-1111, 1000-0111, 1100-0011, 1110-0001, 0100-1011, 0101-1010, 0010-1101 and 01 10-1001



P-228 (1978)

In this issue:



MT 128 September 1989