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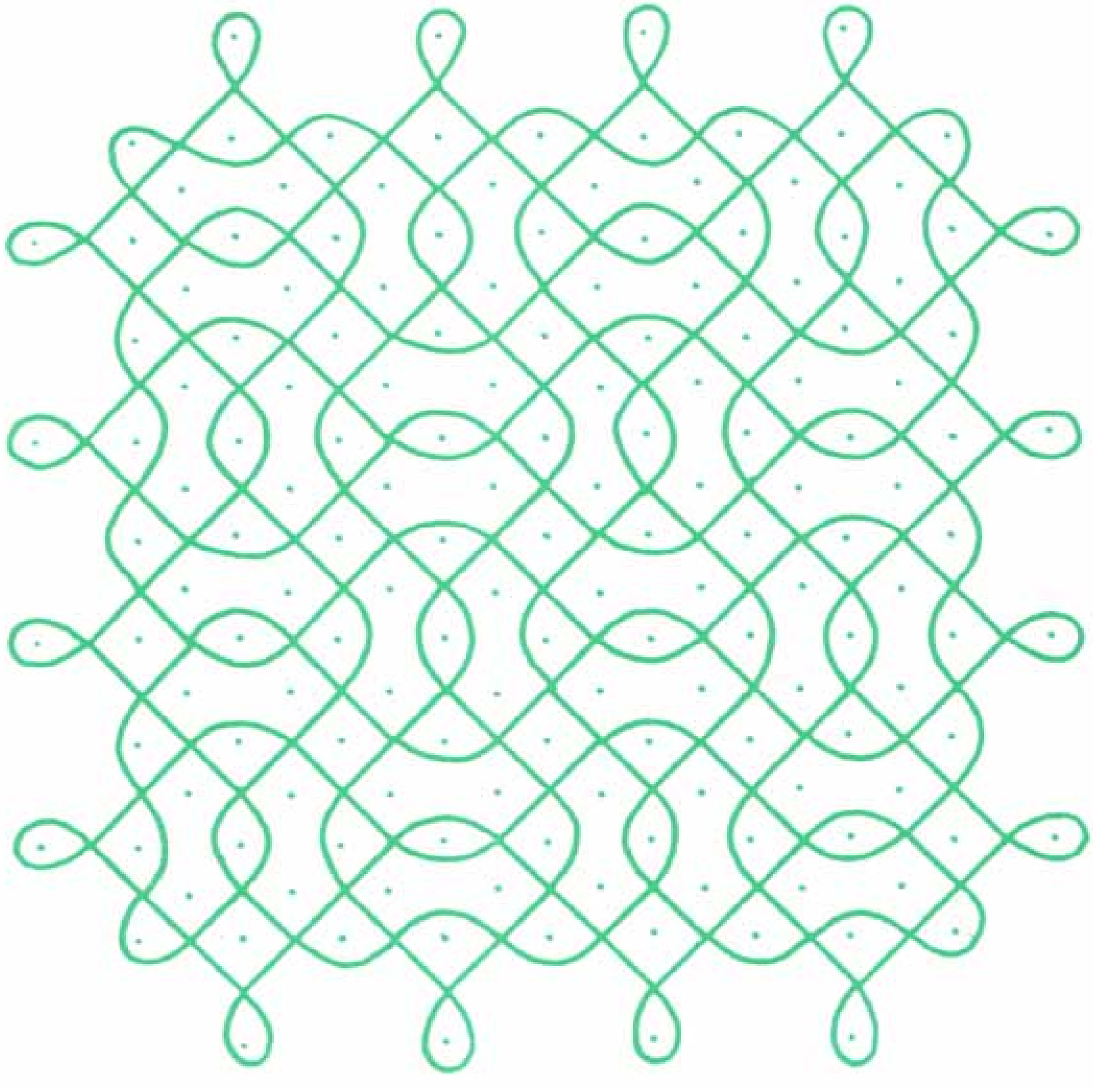

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Illustrations for articles pages 6, 7, 11, 32, 33 by Zena Ginifer.
Parallels: pl7, translated from original Greek by Tetti Pandelias.
Photographs: Stars on pages 30 & 41 from Bretton Woods School, Peterborough.

MATHEMATICS AND MODERN ART: NUMBER INVESTIGATIONS

Ulrich Grevsmühl

Whereas mathematics teachers today are well aware of the fact that mathematics can arouse negative feelings, such as dislike and fear, it sometimes comes as a surprise to them that their colleagues in the arts share problems of a similar nature. The constructive artist Jean Spencer recently expressed this to me in the following way:

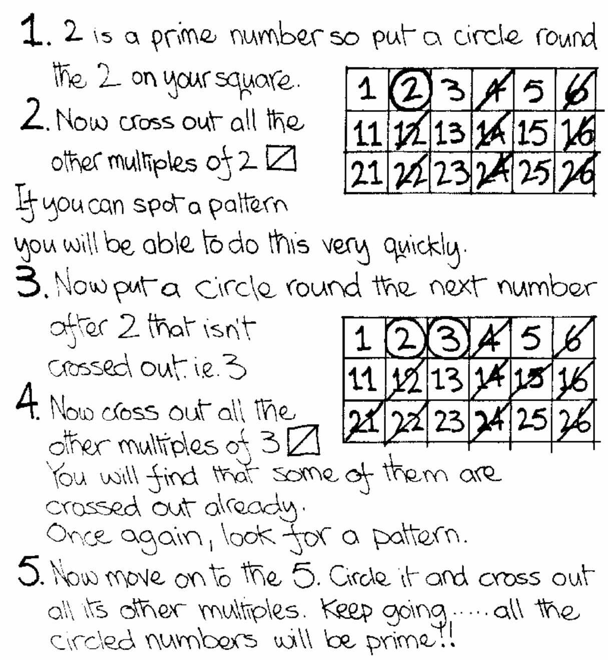
One aspect that 1 am particularly interested in at the moment is to do with the problems naive readers, with no specialist knowledge, have with constructive art. Why‘ when mathematics takes up such a large part of their school curriculum, they find paintings which require the application of what are really very simple logical concepts and operations so difficult, and indeed alienating?

Some of the answers may be found in the way in which both mathematics and art are taught and experienced. Pupils‘ attitudes are greatly affected by the picture the teacher gives of that subject. In teaching, emphasis is laid more often than not on products and subsequent consumption, rather than on the underlying creative processes that lead to them. This is particularly the case when teachers see themselves more in the role of someone who passes on knowledge and skills rather than someone who is engaged in the investigational process of research.

It is important to realize that mathematics and constructive art share a number of common features. They both are a means of communication in that they use the process of abstraction to generate symbolic and visual languages in order to convey certain ideas. Their activities are essentially of a problem solving nature and give rise to the creation of many investigational procedures [1]. Bridging the gap between these two areas is one of the main aims of this series of articles.

Rune Mields *Born in Münster! Westfalia, 1935. Lives in Gologne.*

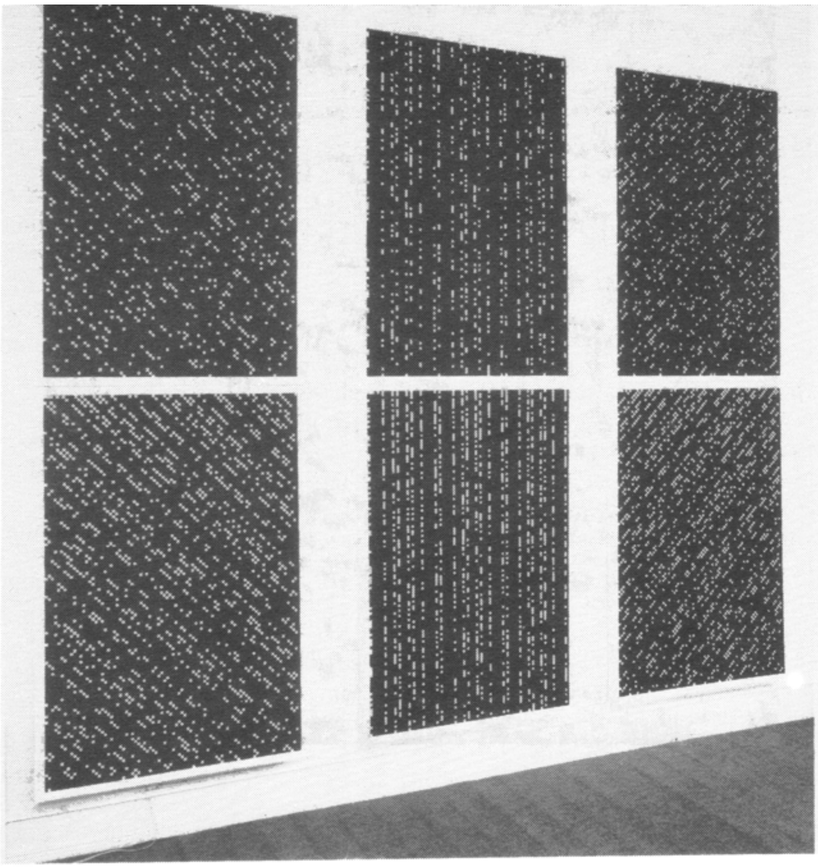
Prime numbers play an important part in the works of Rune Mields. Various forms of representation are used in order to reveal structure and provide insight. In her series of works *The Sieve of Eratosthenes III* (ink on canvas, 280 x 95, 96, 97cms 1977) she applies a famous procedure, named after a Greek mathematician in the third century BC, for generating prime numbers as follows:



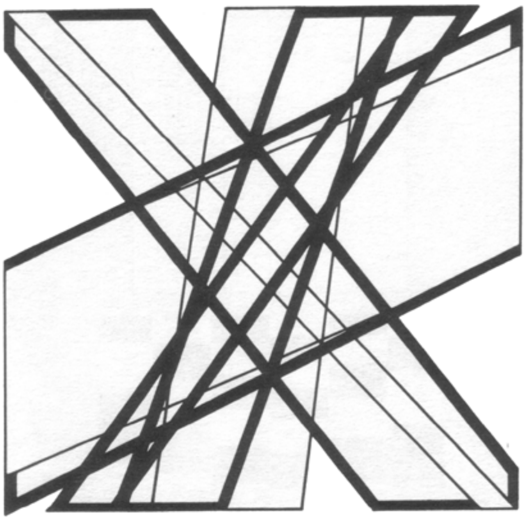
Arranging the number sequence in the form of a table with row sizes 89, 90 and 91, where each prime number is depicted as a small white square, she finds that, depending on the size of the rows different visual structures appear. For the prime number 89 (bottom left corner) the structure is doubly diagonal, for the even number 90 (middle bottom row) it is vertical and for the odd but divisible number 91 (bottom right corner) it is vertical and doubly diagonal [2]. To verify these results the reader might find it useful to write down all prime numbers in number squares of row sizes 11, 10 and 9.

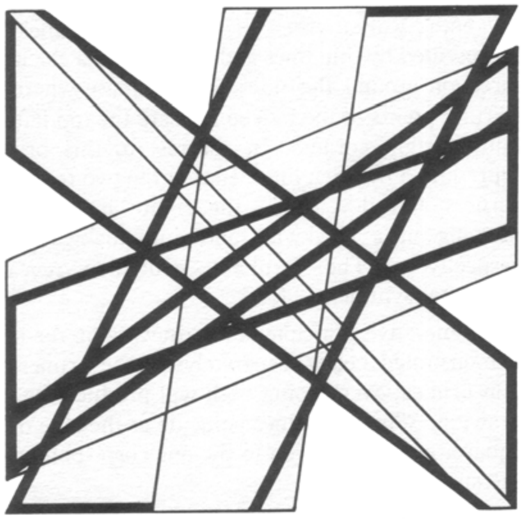
Prime numbers occur less frequently as one moves along the number line so that for very large numbers the structures become indistinguishable. This effect of thinning out of the structures is demonstrated for numbers around 1,000,000 in her three upper works.

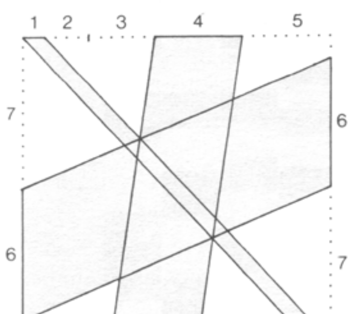
PRIMES

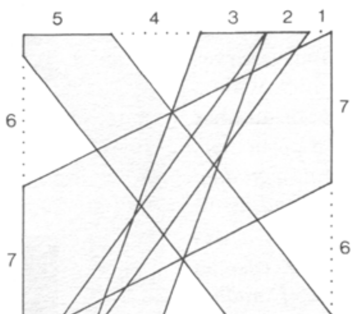


Rune Mields
The Sieve of Eratosthenes III
1977









Malcolm Hughes
Untitled drawings
1986

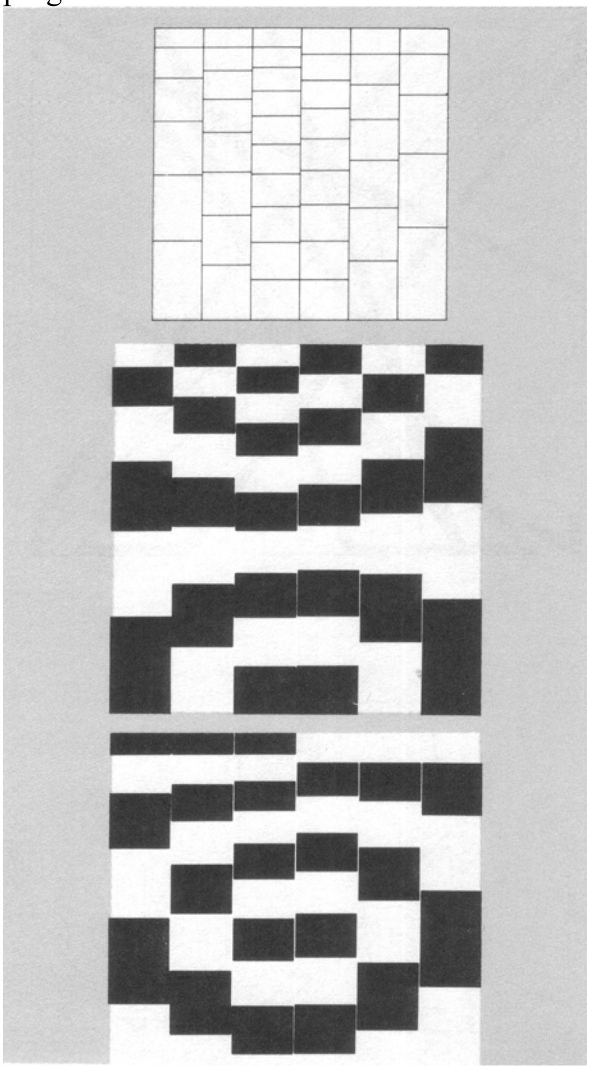
Malcom Hughes *Born in Manchester, 1920. Lives in London.*

For me the making of a drawing is both an act of exploration/analysis (valid in itself and in terms of a particular end product, be it painting, relief/construction, or sculpture) and, perhaps, a psychological necessity — I like the activity..:! [3]
These words by Malcom Hughes apply well to his untitled drawings (1986) where he uses the first natural numbers from 1 to 7 as a basis for investigations. The set of prime numbers (2, 3, 5, 7) and the set of non-primes (1, 4, 6) are the elements with which ordering operations are carried out along the four sides of a square with lengths 14 by progressive addition of the sequence:
1+2+3+4+5+6+7= 28 =4 x 7. He writes:
The primes are for me rather like the wheels of a bicycle, they get one sometimes into unexperienced terrain. The non-primes are like the frame of the bicycle, they keep the whole operation together. The tension generated between the two sets, when both are visually realized, 1 find interesting and stimulating; often this leads to necessary further exploratory work the creative feedback syndrome [3].
In both works, see p35, the non-primes are represented by thin lines and arranged in a clockwise direction around the sides of the square where the starting points of the movements are the top left and bottom right corners. In contrast to this primes, represented by thick lines, start at the two remaining corners and develop in the right- and left-hand drawing in a clockwise and anti- clockwise way, respectively. The resultant structures reveal a rotational symmetry [4].
The generative principle of the drawing on the left is demonstrated below, separately for the primes and non-primes; overlapping both will produce the total drawing. When two lines coincide at the side of the square, priority is given to the one corresponding to the smaller number.

Keith Richardson-Jones *Born in Northampton, 1925. Lives in Tintern, Gwent.*

For Keith Richardson-Jones the vocabulary of geometric form, a term used by Van Doesbury, serves to define a language which provides him with a flexible framework for examining the use of number systems in organizing orders of forms. In particular, arithmetic progressions are used: *to generate a gradual shift of movement or rhythm, up and down and across the plane of the working area. This programme ... gives rise to unexpected diagonal and curvilinear energies within the resultant gestalt. He continues: Paradoxically, the imaginative and intuitive processes are liberated by the predictability of regular number sequences, which take wing when combined or opposed in functions outside of the mathematician's usage.* [5]
In Series 288 24:18 (1975, acrylic on canvas, 80 x 80cms) six contiguous vertical columns are positionally interchanged. Arithmetic progressions mark off each column with horizontal intervals which establish areas and positions of alternate black and white cells. The numerals 5 to 10 designate the columns by referring to the number of cells in each.
The two paintings here and the accompanying drawing (1977, ink/paper 50 x 50cms) share the same column sequence 6/8/10/9/7/5 where the column intervals 2-2-1-2-2. ie the differences between the number of divisions in adjacent columns, are dispersed symmetrically on each side of the vertical centre. The columns 6/8/10 are headed by cells of 18 units (one sixteenth of the total length of the vertical side), the columns 5/7/9 by cells of 24 units (one

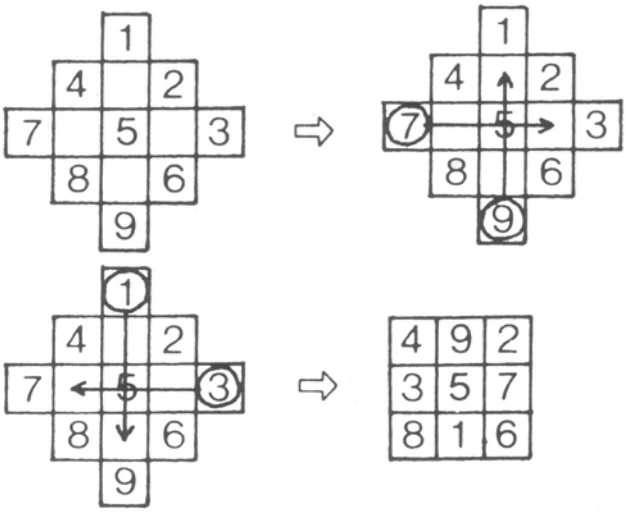
twelfth of the total length of the vertical side) [6].
The total area of all cells in a column comprises 288 units. Using standard formulae for the arithmetic progression you may calculate, for each column, the constant difference of the progression as well as areas and dimensions of the cells.



Susan Tebby *Born in Wakefield, 1944. Lives in Ullesthorpe, Lutterworth*

The systematic constructivist Susan Tebby has extensively investigated patterns of organization by examining the order, structure and visual realization of various mathematical systems where much of her research on geometry design in Roman tessellated pavements has been directly applicable to the development of drawings and constructions [7].
All her sets of drawings ultimately result in 3D reliefs. It is important to her that the transition into three dimensions is not simply a representation, repetition or rearrangement of 2D but a genuine development which contributes something new and significant to the original generative system or organization. Specifically, it is a contribution which only 3D is capable of making and thus it is unique to the development [8].
Her following works deal with some of the aspects of the perceptual development of magie squares from which several series of drawings and reliefs have originated. For the generation of magie squares of odd orders she initially used a construction method, outlined in a book by

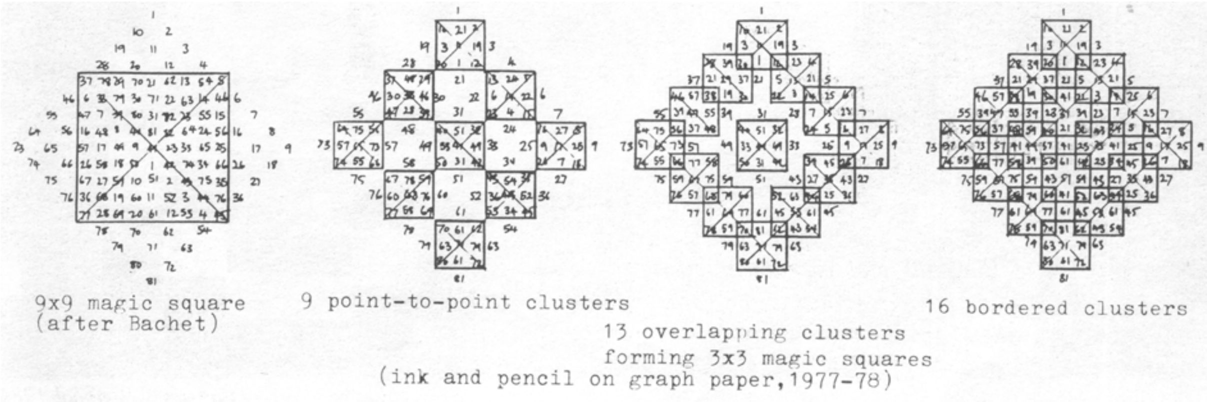
Bächet de Mäiriae [9], which consists of systematic re-arrangements of numbers in diamond formation. A demonstration of this procedure, where the numbers outside the proposed square are transposed into the vaeant cells directly opposite, is given here for a magie square of order 3 [7]:

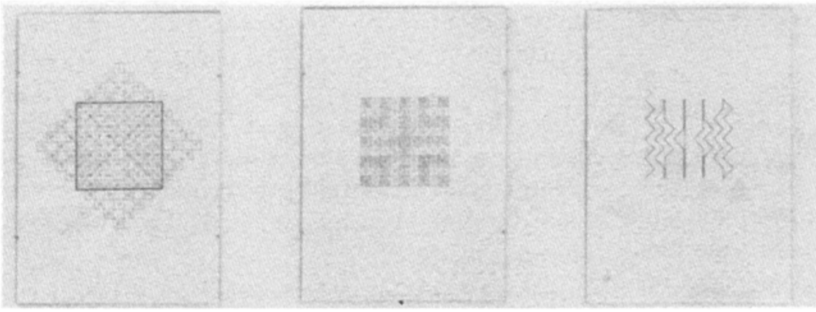
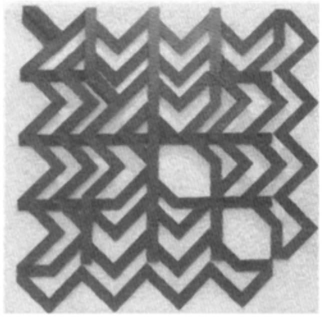


The procedure cannot only be applied to the construetion of magie squares of higher odd order hut also to the subdivision of larger magie squares into smaller ones where the requirement is that every number has to be used at least once.
You may be interested in producing other magie squares of odd order and consider different combinations of subdivisions into magie squares of, say, orders 3 and 5 as well as the relationships of their magie numbers to each other, ie the sums of a row, column or diagonal.
Nine-Point Lattices (1978-83) have been developed as a result of the analysis of 9 x 9 magie squares, parts of which are the series Zigzag (1978), Gross (1978) and Octagon (1980-81) where the total magie square or 5 x 5 subdivisions of it are considered. The pattern of even and odd numbers in these systems is represented by white and grey areas, respectively.
Vertical line drawings are produced to record the change in variable for each column according to the following rules:

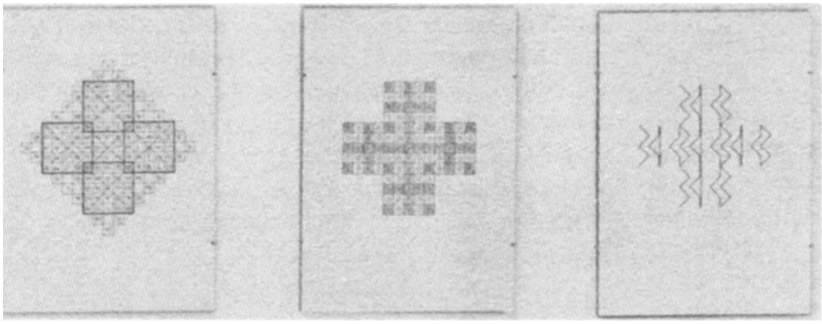
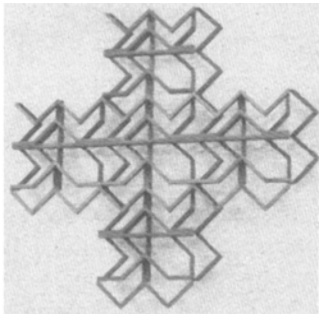
The pattern for the horizontal rows may be found in a similar way and results from a clock wide quarter turn of the vertical line drawings about the centre of the magie square. The final stage of the development combines both columnar and row patterns into a sculptural construction.
For the translation of the mathematical structure into visual representation it is often necessary to reduce the number of variables, not just for reasons of practicality, hut also to amplify certain aspects and structures of the system which are visually more significant and which can be developed as an artifact. Modular reduction is frequently applied in the investigation of magie squares, as for Nine-Point Lattices where modulo 2 produces a pattern of oddness/eveness.
Similarly, in Lattices, Constructive Gates for The International Garden Festival in Liverpool, 1984 (alkyd lacquer on Finnish birch. 4‘6“ high, 8 fl. span, 5“ deep) a magie square of order 14 has been reduced by modulo 13. Visual realization is achieved again by representing the change in oddness/evenness for each of the 14 columns in the same fashion as above. When

the gates are open, they face each other as a two-fold mirror image reflection.

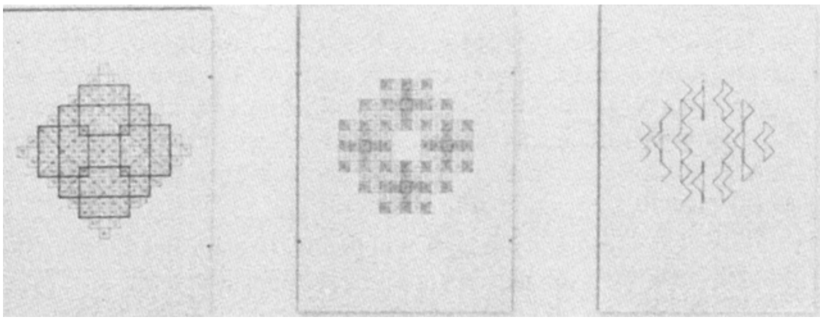
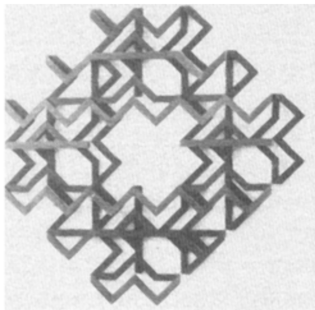


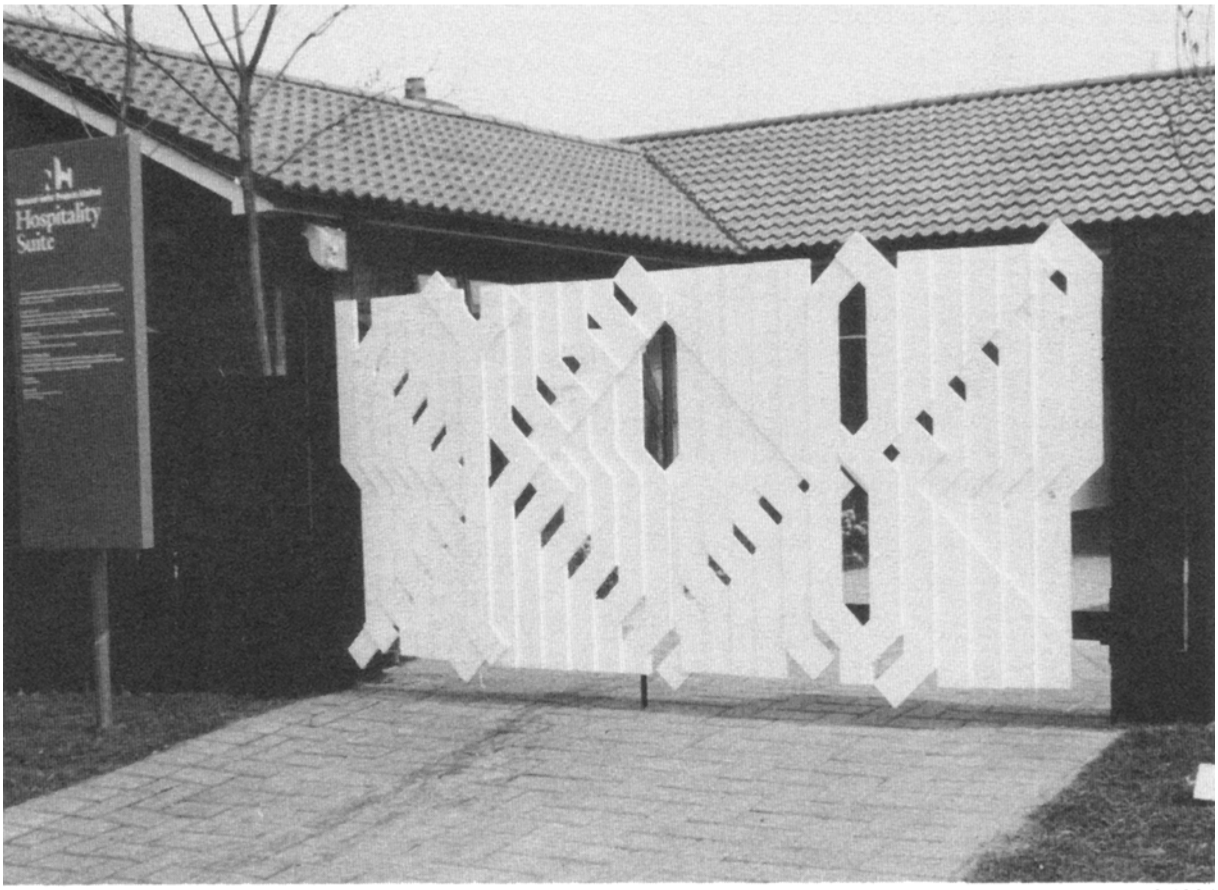
Zigzag

Cross

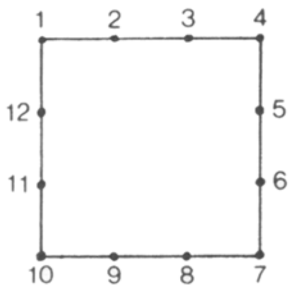
Octagon

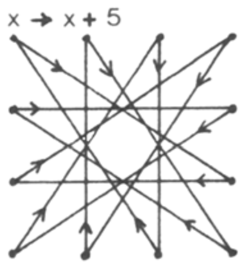


Lattices, Liverpool 1984

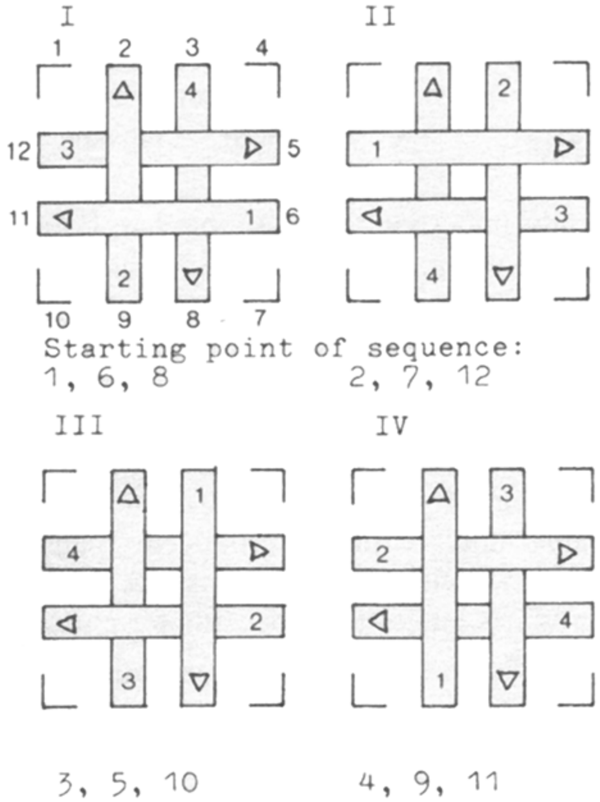
Nathan Cohen *Born in London, 1962. Lives in London*

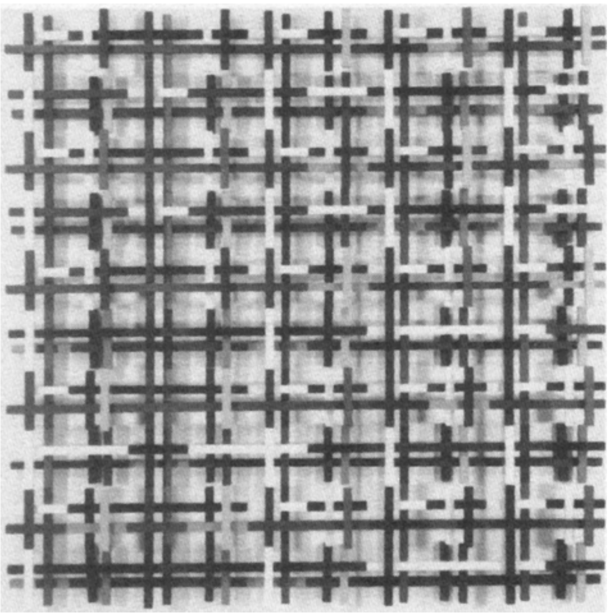
Nathan Cohen has primarily been concerned with the investigation of closed mathematical systems and the search for complex, non-repetitive images which can be deduced from these and translated visually in the form of reliefs. The following system shows the complexity of image that can be generated by a relatively simple numeric ordering and permutative repetition of elements [10].
The sides of a square are divided equally into three parts, and corner and nodal points are numbered clockwise 1 to 12. Cyclic mappings of the form $x \rightarrow x + n$, $n \in \{0,1,2...,11\}$ are used to connect the points by straight lines. The only non-trivial solutions which connect all points 1 to 12 by a single closed path are the sequences $x \rightarrow x + 5$ and $x \rightarrow x + 7$:





(The mapping $x \rightarrow x + 7$ produces the same pattern as. $x \rightarrow x + 5$ with reversed direction of orientation.) In each of these sequences four elements are generated, two horizontals and two verticals, which correspond directly to colored rods in the subsequent construction of the reliefs. The ordering of these elements depends on the starting point, 12 in all, from which the sequence emerges. For the sequence x → x + 5 there are four different possibilities:





United Series B10/No.1986 (painted mahagony, 177.8 x 177.8 x 17,8 cms)

The cells 1 to IV form the structural basis in the construction of quadratic reliefs such that elements of each cell connect with those of the cells immediately adjacent to form the overall structure In particular, even and odd rows show an alternating pattern of I-III-I-... and Iv-II-Iv-... ordering, respectively. The reliefs consist of permutative orderings of the 12 starting points, each of which is linked to one of the four cells where the color of each of the four elements is determined by a certain permutative code.

Pädagogische Hochschule Freiburg, West Germany

I am indebted to N Cohcn, M Hughes, K Richardson-Jones and S Tebby for their collaboration. Also to the gallery Jöllenbeck, Gologne, for supplying materials of R Mields.

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6 ---, serial works 1974 column series 288
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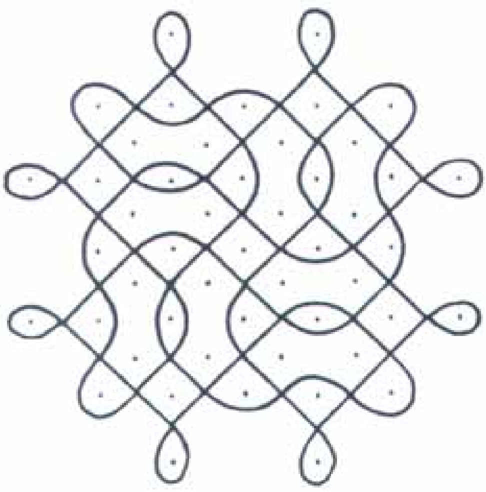

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